ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

SENSITIVITY OR TEMPERATURE FIELD IN THE SYSTEM PROTECTIVE CLOTHING – FOREARM WITH RESPECT TO PERTURBATIONS OF EXTERNAL HEATING CONDITIONS

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Keywords: Bio-heat transfer, Protective clothing, Sensitivity analysis, Control Volume Method, Numerical simulations.

Abstract. *In the paper the problem concerning the numerical modeling of thermal processes in domain of biological tissue secured by a layer of protective clothing being in the thermal contact with the environment is discussed. The cross-section of the forearm (2D problem) is treated as the non-homogeneous domain in which the sub-domains of skin tissue, fat, muscle, bone and blood vessels are distinguished. Between skin tissue and protective clothing the air gap is taken into account. This sub-domain is treated as a solid body which thermal conductivity is defined in the special way. The process of external heating is determined by the Neumann boundary condition and the sensitivity analysis with respect to the perturbations of the boundary heat flux is discussed. Both the basic boundary-initial problem and the sensitivity one are solved by means of the control volume method using the Voronoi polygons.*

1 INTRODUCTION

The problem of skin tissue heating including the layer of protective clothing is described by the system of partial differential equations (energy equations), the boundary condition given on the external surface of the system, the boundary conditions between skin tissue and protective clothing, the boundary conditions on the surfaces limiting the successive sub-domains of forearm and the initial conditions. In the version presented in this paper the air gap between fabric and skin tissue is treated as a solid body (see: next chapter) and on its boundaries the continuity of temperature field and heat fluxes are assumed.

The transient temperature field in the tissue sub-domains is determined by the Fourier-type equation called the Pennes equation [1-7]. This equation contains two additional components (the source functions), this means the perfusion heat source and the metabolic heat source. The Pennes equation belongs to the group of macroscopic tissue models. In the recent years the others model based on the Cattaneo-Vernotte equation (e.g. [8]) or the dual phase lag equation (e.g. [9]) appeared, but the Pennes approach is, so far, most commonly used. The domain considered (a section of forearm – 2D problem) is the non-homogeneous one and constitutes a composition of skin tissue, fat, muscle, bone and blood vessels (arteries and veins). The successive sub-domains differ in the values of thermal parameters, in this paper the data taken from [10] are applied. Temperature fields in the domains of fabric and trapped air are described by the well known Fourier equation (diffusion equation). The thermophysical parameters of textiles can be found in [11].

At the stage of numerical computations the Control Volume Method (CVM) is applied, in other words, the domain considered is divided into a certain number of small cells and the governing equations in the integral form are used individually to each one of them. This procedure guarantees, a priori, the conservation of physical quantities like mass, momentum and energy, is extremely flexible and conceptually simple. In this paper, the 2D control volumes correspond to the Voronoi polygons (also called the Thiessen or Dirichlet cells in two dimensions) have been used. Such a version of CVM was in details discussed by Ciesielski and Mochnacki in [12].

The sensitivity analysis presented here concerns the changes of transient temperature field in domain considered due to the perturbations of the boundary heat flux determined by the Neumann boundary condition given on the external surface of the system. The sensitivity model is created by the differentiation of energy equations and boundary-initial conditions with respect to the parameter considered (a direct approach, e.g. [13-16]).

2 MATHEMATICAL DESCRIPTION OF THE PROCESS

The domain considered (cross section of the forearm middle part [17]) is shown in Figure 1.

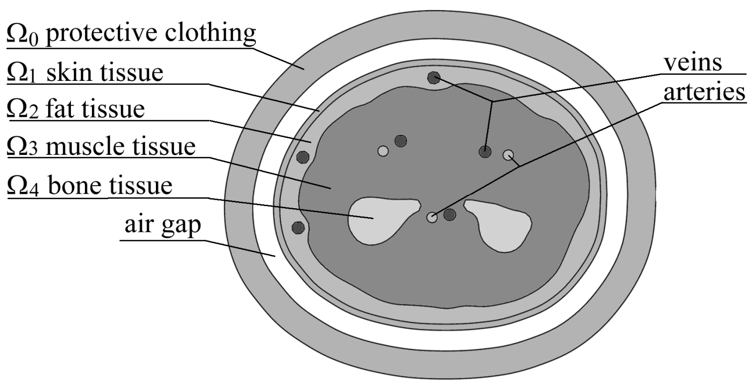

Figure 1: Simplified 2D geometrical model of the forearm with protective clothing cross-section.

Heat transfer processes proceeding in the tissue volume are described by the system of the Pennes equations of the form

$$c_e\left(T\right)\frac{\partial T_e\left(x,t\right)}{\partial t}=\nabla\left[\lambda_e\left(T\right)\nabla T_e\left(x,t\right)\right]+Q_{per\,e}\left(x,t\right)+Q_{met\,e}\left(x,t\right),\qquad e=1,\ldots,4 \tag{1}$$

where e = 1, 2, 3, 4 refers to the tissue sub-domains (skin, fat, muscle and bone, respectively), c_e is the volumetric specific heat, λ_e is the thermal conductivity, Q_{per} and Q_{met} [W/m^3] are the capacities of volumetric internal heat sources connected with the blood perfusion and metabolism, T, $x = \{x_1, x_2\}$, t denote the temperature, spatial co-ordinates and time. The perfusion heat source is given by the formula

$$Q_{per\,e}\left(x,t\right)=c_b G_{be}\left(T\right)\left[T_b-T_e\left(x,t\right)\right],\qquad T_b=\left(T_{b\,artery}+T_{b\,vein}\right)/2 \tag{2}$$

where G_{be} is the blood perfusion [m$^3_{\text{blood}}$/(s m$^3_{\text{tissue}}$)], c_b is the blood volumetric specific heat and $T_{b\ artery}$ and $T_{b\ vein}$ are the arterial and vein blood temperatures. Metabolic heat source Q_{met} (x, t) can be treated as a constant value.

Equation describing the transient temperature field in the domain of fabric is of the form

$$c_0\left(T\right)\frac{\partial T_0\left(x,t\right)}{\partial t}=\nabla\left[\lambda_0\left(T\right)\nabla T_0\left(x,t\right)\right] \tag{3}$$

The similar equation with the parameters corresponding to the properties of air describes the temperature field in the domain of trapped air between fabric and forearm.

So, the layer of trapped air is treated as a solid body (a convection in this sub-domain can be neglected). Through the air gap heat flows by the radiation and conduction, in particular

$$q_r(x,t)=\alpha_r\left[T_F(x,t)-T_T(x,t)\right],\qquad q_c(x,t)=\frac{\lambda_a}{\delta}\left[T_F(x,t)-T_T(x,t)\right] \tag{4}$$

where α_r is the radial heat transfer coefficient, λ_a is the air thermal conductivity, δ is the air gap thickness, T_F, T_T are the averaged temporary temperatures of fabric and skin surfaces. As is well known

$$\alpha_r=10^{-4}\varepsilon_{F-T}C_c(T_F+T_T+546)\left[\left(\frac{T_F+273}{100}\right)^2+\left(\frac{T_T+273}{100}\right)^2\right] \tag{5}$$

and

$$\frac{1}{\varepsilon_{F-T}}=\frac{1}{\varepsilon_F}+\frac{1}{\varepsilon_T}-1 \tag{6}$$

is the substitute emissivity, while $\varepsilon_F, \varepsilon_T$ are the emissivities of fabric and skin tissue surfaces and C_c = 5.67 W/m^{2}K^2.

The total heat flux exchanged between the surfaces is equal to

$$q(x,t)=\alpha_r\left[T_F(x,t)-T_T(x,t)\right]+\frac{\lambda_a}{\delta}\left[T_F(x,t)-T_T(x,t)\right]=(\alpha_r+\frac{\lambda_a}{\delta})\left[T_F(x,t)-T_T(x,t)\right] \tag{7}$$

Let us introduce the substitute air thermal conductivity

$$q(x,t)=\frac{\lambda_{sub}}{\delta}\left[T_F(x,t)-T_T(x,t)\right]=(\alpha_r+\frac{\lambda_a}{\delta})\left[T_F(x,t)-T_T(x,t)\right] \tag{8}$$

and finally

$$\lambda_{sub} = \delta(\alpha_r + \frac{\lambda_a}{\delta}) \tag{9}$$

It should be pointed out, that the volumetric specific heat of trapped air corresponds to the temperature-dependent values which can be found in literature (e.g. [18]).

On the contact surface between the tissue sub-domains, the continuity of temperature and heat fluxes are assumed

$$x \in \Gamma_{e,e+1}: \begin{cases} -\lambda_e \dfrac{\partial T_e(x,t)}{\partial n} = -\lambda_{e+1} \dfrac{\partial T_{e+1}(x,t)}{\partial n} \\ T_e(x,t) = T_{e+1}(x,t) \end{cases}, \qquad e = 1,2,3 \tag{10}$$

where $\partial / \partial n$ is a temperature derivative in normal direction. The same condition is accepted on the surfaces limiting the air gap.

On the outer surface of the fabric, the combined Robin and Neumann boundary conditions are taken into account

$$x \in \Gamma_{out}: \quad -\lambda_0 \frac{\partial T_0(x,t)}{\partial n} = -\alpha_{out}\left(T_{amb} - T_0(x,t)\right) + \begin{cases} -aq_b, & \text{if } t \in (0, t_{heating}] \\ 0, & \text{if } t \in (t_{heating}, t_{final}] \end{cases} \tag{11}$$

where q_b is the boundary heat flux, a is the absorptivity of the outer surface of the protective clothing, α_{out} is the heat transfer coefficient and T_{amb} is the ambient temperature. On the surfaces between the blood vessels and soft tissue sub-domains the Robin condition is taken into account, at the same time the heat transfer coefficients and the arterial and vein blood temperatures are assumed to be known.

The initial conditions are also given

$$t = 0: \quad T_e(x,t) = T_{steady}(x), \qquad e = 0,1,\ldots,4 \tag{12}$$

where T_{steady} is the temperature distribution corresponding to the steady state conditions in the tissue-fabric domain for the given initial ambient temperature and the initial external heat transfer coefficient.

3 SENSITIVITY MODEL

The sensitivity model presented in this paper concerns the changes of transient temperature field in domain considered due to the perturbations of the boundary heat flux. The equations creating the sensitivity model have been obtained by the differentiation of the basic equations and conditions with respect to q_b.

Let us denote the sensitivity function $\partial T(x,t)/\partial q_b$ by $U(x, t)$. Then the sensitivity equations for the tissue sub-domains are of the form

$$c_e \frac{\partial U_e(x,t)}{\partial t} = \lambda_e \nabla^2 U_e(x,t) - c_b G_{be} U_e(x,t), \qquad e = 1,\ldots,4 \tag{13}$$

while for the fabric

$$c_0 \frac{\partial U_0(x,t)}{\partial t} = \lambda_0 \nabla^2 U_0(x,t) \tag{14}$$

One can see that the above equations concern the constant values of fabric and tissue thermophysical parameters (the authors do not dispose the convincing information connected with the temperature-dependent relations).

For the trapped air the sensitivity equation is the following

$$c_a \frac{\partial U_a(x,t)}{\partial t} + \frac{\mathrm{d}c_a}{\mathrm{d}T} U_a(x,t) \frac{\partial T_a(x,t)}{\partial t} = \nabla\left[\lambda_a \nabla U_a(x,t)\right] + \frac{\mathrm{d}\lambda_a}{\mathrm{d}T} \nabla\left[U_a(x,t) \nabla T_a(x,t)\right] \tag{15}$$

On the external surface the boundary condition has a form

$$x \in \Gamma_{out}: \quad -\lambda_0 \frac{\partial U_0(x,t)}{\partial n} = \alpha_{out} U_0(x,t) + \begin{cases} -a, & \text{if } t \in (0, t_{heating}] \\ 0, & \text{if } t \in (t_{heating}, t_{final}] \end{cases} \tag{16}$$

'Rebuilding' of the others boundary and initial conditions is rather simple and it will not be discussed here.

4 RESULTS OF COMPUTATIONS

In Figure 2 an example of the control volume mesh covering the sub-domains considered is shown. Here, the domain being a composition of the forearm, protective clothing and air gap is divided into 3429 control volumes.

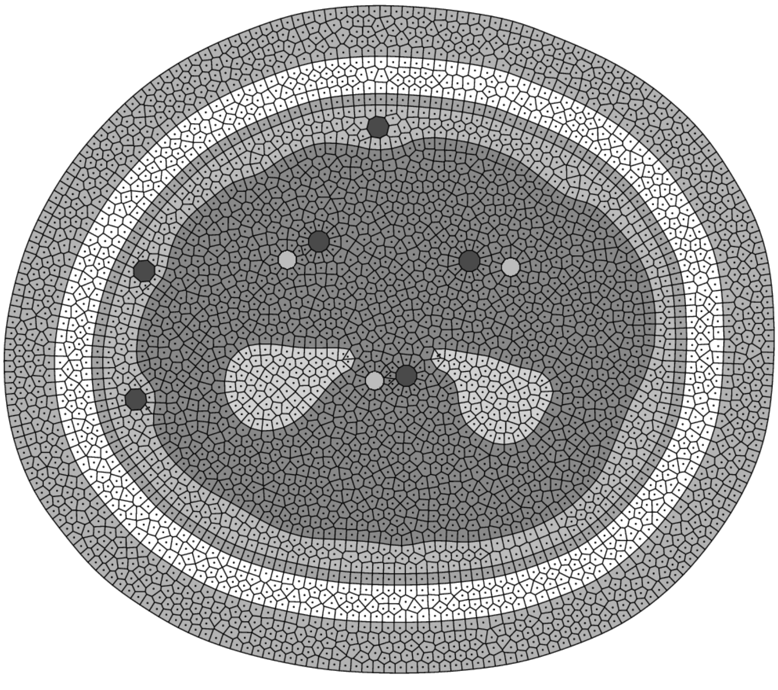

Figure 2: Control volume mesh of the considered sub-domains for the cross-section of the forearm and protective clothing.

The initial temperature distribution of the fabric-forearm system (see: Figure 3) corresponds to the steady state conditions. They are found for the air temperature equals T_{amb} = 20 ^{o}C, and heat transfer coefficient α_{out} = 5 W/m^{2}K. Additionally the following blood temperatures are assumed: $T_{b\ artery}$ = 36 ^{o}C, $T_{b\ vein}$ = 35 ^{o}C, while $\alpha_{artery} = \alpha_{vein}$ = 5000 W/m^{2}K.

In the numerical simulation at the moment t = 0 the process of heating starts (q_b = 2100 W/m^2, a = 0.3, while T_{amb} = 30 ^{o}C and α_{out} = 5 W/m^{2}K) and this process is continued until $t_{heating}$ = 3 min. Next, the cooling process takes place (q_b = 0 W/m^2, T_{amb} = 30 ^{o}C and α_{out} = 5 W/m^{2}K).

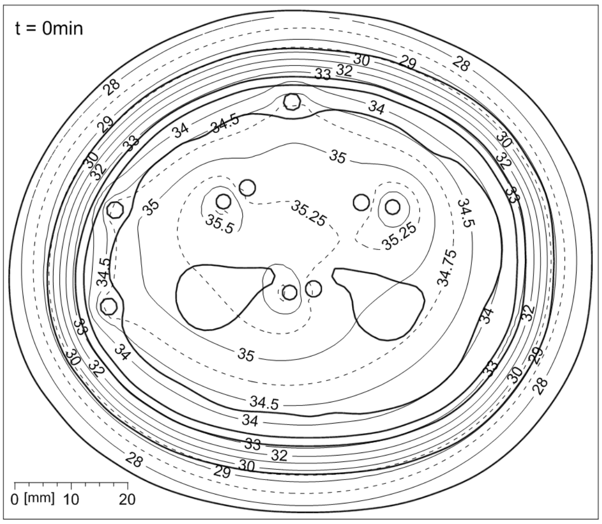

Figure 3: The initial temperature distribution.

The basic solution in the form of heating curves representing the average temperature in the layers of skin, the inner and outer layers of protective clothing is shown in Figure 4. The next Figure presents the temporary solutions for times 3 min. and 6 min.

In Figures 6 and 7 the numerical solution of sensitivity problem is shown. In particular, Figure 6 illustrates the changes of sensitivity function $U_e(x,t) = \partial T_e(x,t) / \partial q_b$ (also representing the average values in the skin layer, inner and outer layers of protective clothing). The temporary distributions of sensitivity with respect to perturbation of the boundary heat flux are shown in Figure 7.

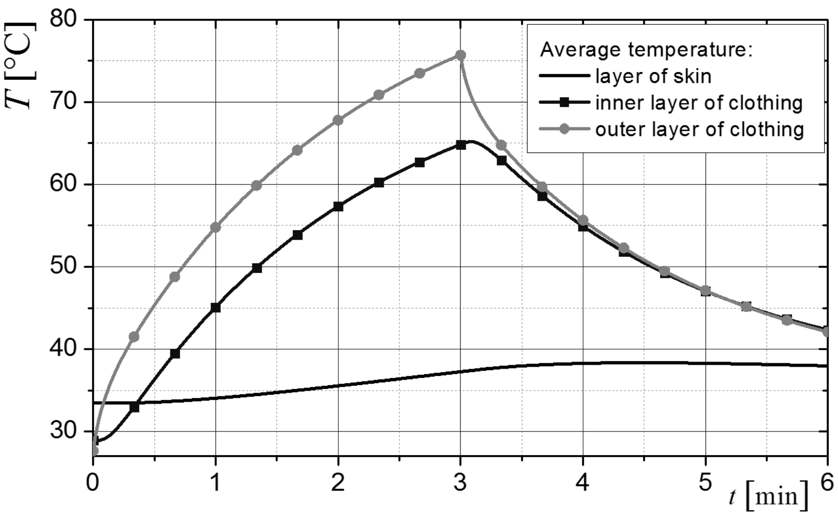

Figure 4: Heating curves representing the average temperature in the skin layer, inner and outer layers of protective clothing.

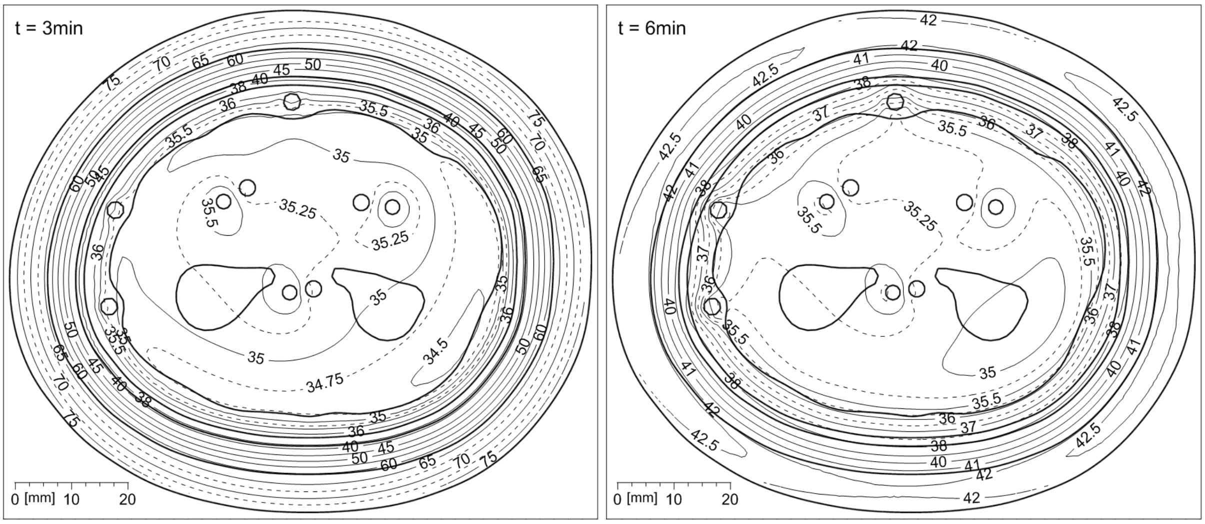

Figure 5: Temporary solutions for 3 and 6 min.

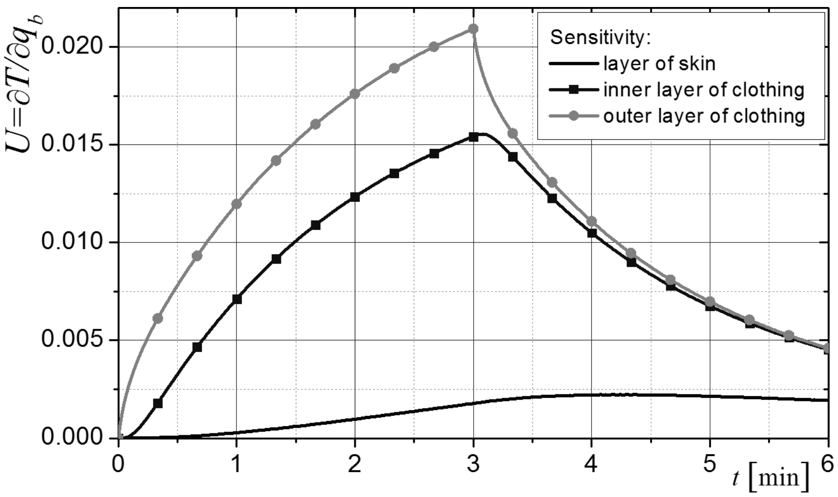

Figure 6: Sensitivity with respect to the boundary heat flux.

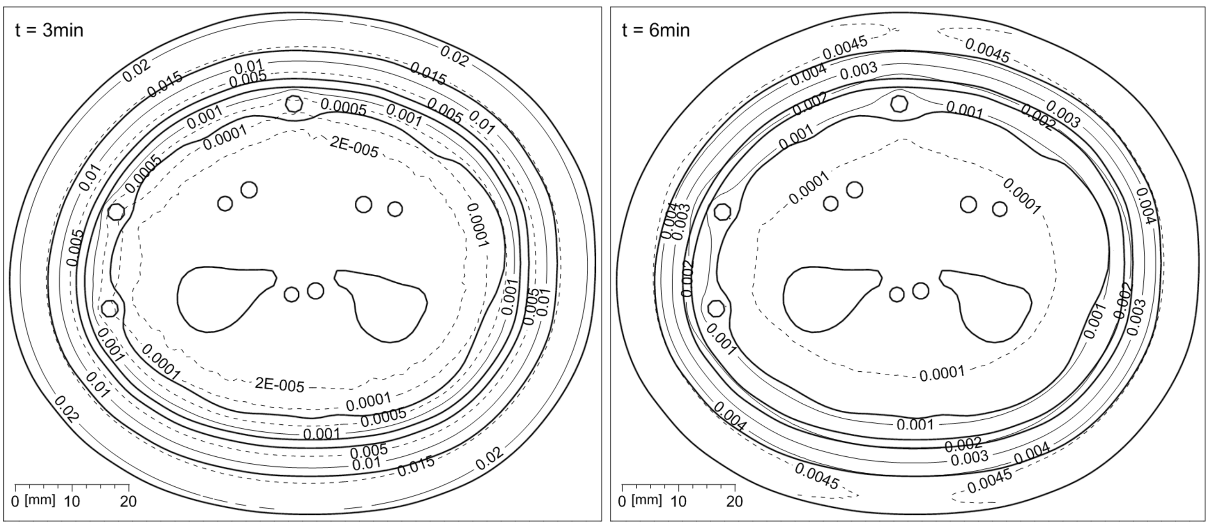

Figure 7: Distribution of sensitivity with respect to the boundary heat flux for times 3 and 6 min.

5 CONCLUSIONS

In this paper the numerical model of thermal processes occurring in the tissue-fabric domain exposed to an external heating by the strong boundary heat flux have been discussed. The computations have been performed on the basis of the author's variant of CVM using the Voronoi polygons. Such a method can be easily adapted to the needs of bio-heat transfer problems and it is an effective approach to the numerical modeling of thermal processes proceeding in the domain of biological tissue. Moreover, a discretization of 2D domain of the complex heterogeneous shape using the Voronoi polygons has many advantages, among others, the shapes of sub-domains can be exactly reconstructed. The CVM assures the correct modeling of the energy balances and the implementation of the different types of boundary conditions is simple.

The analysis of the results presented in this paper, gives a number of the significant information. The increase of temperature in the fabric domain proceeds essentially faster in comparison with the tissue domain. In the case of accidental contact between hot fabric and human body the tissue burns can take place. One can note that at the final moment of the heating process, the temperature of the protective clothing is significantly higher than the temperature of the skin layer and at the stage of the cooling of outer fabric surface, the heat is still transferred to the forearm domain. In order to achieve the thermal comfort of a person during the heating process (the maximum temperature of the skin tissue layer should not exceed the value more than 38 °C) the adequate period of the heating time in the presented simulation has been chosen. Such an analysis of results allows one to the prediction of a situation which is not preferable by a person. Further analysis allows one to determine the maximum stay time at the given heat flux for a person under secure conditions.

In this work the sensitivity analysis with respect to the perturbations of the boundary heat flux is considered and one can formulate the following conclusions:

- the maximum value reaches at the final stage of heating and the most essential changes are in the outer layer of fabric,
- after period of the heating process, the sensitivity rapidly decreases,
- during the considered simulation time in the tissue domain the sensitivity is practically close to zero.

ACKNOWLEDGEMENT

The paper has been sponsored by WSZOP Katowice (Research Project No. 3/2013).

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