Discrete gradient theorem and element-based integration in meshless methods

Guillaume Pierrot[1*], Gabriel Fougeron[1, 2]

[1]ESI Group
99 rue des Solets BP80112, 94513 Rungis cedex, France
guillaume.pierrot@esi-group.com

[2]Centrale-Supélec
Laboratoire MSSMat - UMR CNRS 8579
Grande Voie des Vignes 92290, Châtenay-Malabry, France
gabriel.fougeron@esi-group.com

ABSTRACT

The *gradient theorem*, which relates the volume integral of a gradient field to the surface integral of its scalar potential is one of the fundamental theorems of vector calculus. Whereas this property is automatically passed to the discrete level for mesh-based methods due to the exactness of test functions integration it is usually not the case for meshless methods.

The importance of fulfilling a discrete version of it when solving the Poisson equation associated with an incompressible Navier-Stokes solver, however, has been acknowledged by many authors.

In [2] by example the role of such a discrete analog of the gradient theorem in passing the so-called patch test and recovering optimal convergence rate is demonstrated and a correction procedure on the gradient coefficients that involves the solving of a global linear system is proposed, whereas in [1], the framework is extended to Petrov-Galerkin formulations and the property is recast as *variational consistency*.

In a recent communication [3] the authors have proposed a new light on the question, introducing in particular the concept of approximate compatibility in the context of nodal integration. This *relaxed requirement* allows the saving of numerous iterations in the solving of the correction system while maintaining an optimal convergence rate. As an alternative to the correction of gradient coefficients, one might play on the nodal arrangement itself to recover a discrete gradient theorem, as investigated in an associated paper [4].

In this communication the analysis introduced in [3] will be extended to the context of element-based integration for which more compact stencils are obtained as well as more freedom in the setting of the correction procedure. At first, the concept of element will be discussed in the framework of meshless methods, then several correction procedures will be presented and their efficiency compared. Emphasis will be put in particular on a *one-shot* approach for which both the correction and the Poisson systems are solved in the same time, hence avoiding the *over-correction* of gradient coefficients and thus allowing the saving of significant CPU time.

References

[1] *Chen, J. S., Hillman, M., Rüter, M.*, “An Arbitrary Order Variationally Consistent Integration Method for Galerkin Meshfree Methods”, Int. J. Numer. Meth. in Engng, 95, 387–418 (2013)
[2] *Bonet J., Kulasegaram S.*, “Correction and stabilization of smooth particle hydrodynamics methods with applications in metal forming simulations”, Int. J. Numer. Meth. Engng., 47, 1189-1214 (2000)
[3] *Pierrot G., Fougeron G.*, “Enforcing differentiation/integration compatibility in meshless methods with nodal integration”, Particles 2015 conference, Barcelona, Spain, (28-30 september 2015)
[4] *Fougeron G., Pierrot G.*, “Recovery of differentiation/integration compatibility of meshless operators via local adaptation of the point cloud in the context of nodal integration”, ECCOMAS Congress 2016, Crete Island, Greece, (5-10 june 2016)