ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

GEARBOX DESIGN VIA MIXED-INTEGER PROGRAMMING

Bastian Dörig[1], Thorsten Ederer[1,2], Peter F. Pelz[2], Marc E. Pfetsch[1], and Jan Wolf[3]

[1] Department of Mathematics, Technische Universität Darmstadt
Dolivostr. 15, 64293 Darmstadt, Germany
e-mail: {doerig, ederer, pfetsch}@mathematik.tu-darmstadt.de

[2] Department of Mechanical Engineering, Technische Universität Darmstadt
Otto-Berndt-Str. 2, 64287 Darmstadt, Germany
e-mail: {thorsten.ederer, peter.pelz}@fst.tu-darmstadt.de

[3] Department of Economics, Universität Siegen
Hölderlinstr. 3, 57076 Siegen, Germany
e-mail: jan.wolf@uni-siegen.de

Keywords: Mixed-Integer Nonlinear Programming, Technical Operations Research, Global Optimization, Gearbox Design, Mechanical Transmission System.

Abstract. *Gearboxes are mechanical transmission systems that consist of multiple gear wheels and shafts. The transmission ratios of the gears are determined by the size and interconnection of these components. Gearboxes have to adhere to very strict requirements regarding weight, production costs, and available space. Moreover, the load cases as a result of motor-vehicle-pairings are often not clear a priori. Therefore, automobile manufacturers are confronted with a multicriteria design problem under uncertainty.*

In this work, we present an approach on how to formulate the gearbox design problem as a mixed-integer nonlinear program. This enables us to compute provably globally optimal gearbox designs. We show how different degrees of freedom, input parameters and the numerical accuracy influence the computation time and the quality of solutions.

Acknowledgement. The authors thank the German Research Foundation DFG for funding this research within the Collaborative Research Center SFB 805 "Control of Uncertainties in Load-Carrying Structures in Mechanical Engineering".

1 INTRODUCTION

Gearboxes are mechanical transmission systems that consist of multiple gear wheels and shafts, see Figure 1. The transmission ratios of the gears are determined by the size and interconnection of these components. As central element of the drive train, gearboxes are highly relevant for the efficiency and durability of motor vehicles. They have to adhere to very strict requirements regarding weight, production costs, and available space. Moreover, the load cases as a result of motor-vehicle-pairings are often not clear a priori. Hence, automobile manufacturers are confronted with a multicriteria design problem under uncertainty.

A traditional approach to system design problems is to manually identify and compare a selection of promising proposals. However, the gearbox is a highly complex system. A comparative analysis of every imaginable gearbox topology is doomed to failure in view of the combinatorial explosion. Engineers work around this problem through a bottom-up design approach: Individual components are optimized and combined to subsystems, which in turn are optimized and combined to larger subsystems. This approach usually leads to good designs, but one cannot make any statement about the objective quality of the resulting systems.

Optimization problems in engineering are often tackled with probabilistic methods, e.g. swarm intelligence or genetic algorithms. They are easy to adapt to new problems and often find good solutions in a short amount of time. However, it is unclear how long such heuristics need to keep running. We do not know whether they will be able to find even better solutions or not. In contrast, mathematical optimization methods are capable of finding *provably globally optimal* solutions by quantifying how much potential for improvement remains at any time.

In this paper, we present a mixed-integer nonlinear program (MINLP) that generates gearbox system designs which are provably globally optimal with respect to a given objective. As example gearboxes, we focus on dual-clutch transmission systems. Technical details on the topic of gearboxes can be found e.g. in [1] and [2].

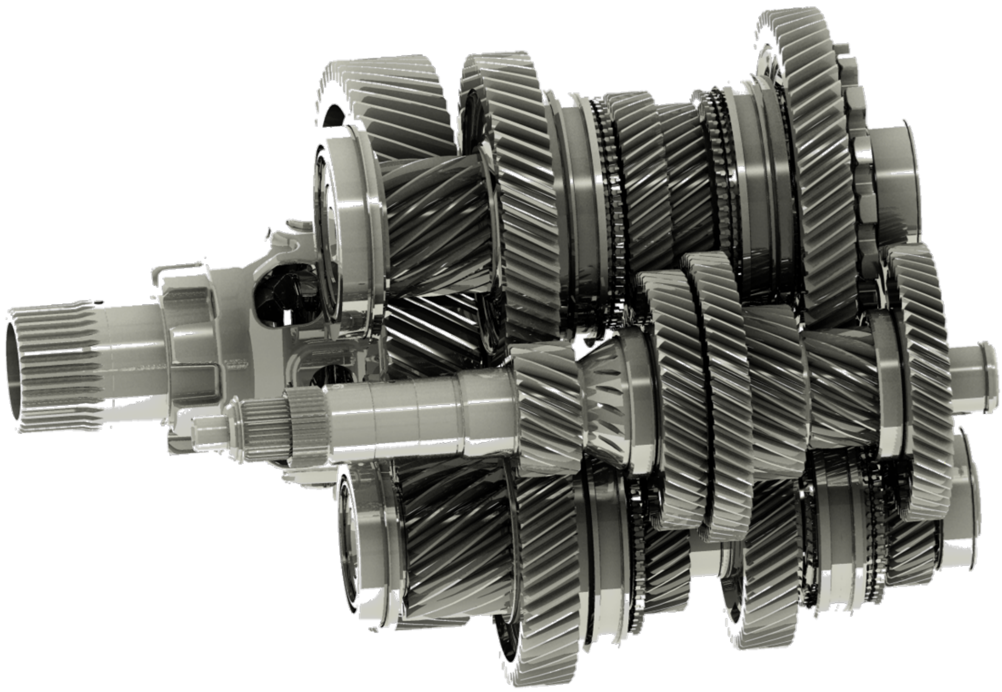


Figure 1: Rendered image of a dual-clutch transmission (Source: Volkswagen AG).

The paper is organized as follows: In Section 2, we introduce the principle of dual-clutch transmissions. In Section 3, we present the mathematical model and discuss possible variations. In Section 4, we show some computed gearboxes and compare runtimes of two MINLP solvers.

2 DUAL-CLUTCH TRANSMISSIONS

The abstract task of a gearbox is to transfer power (given by a torque and an angular velocity) from the input shaft (which is driven by the motor) to the output shaft (which is connected to the differential and moves the wheels). The power flow is established by engaging suitable gear wheels. The transformation ratio of the gear can be adjusted by changing the ratio of the gear wheel radii: an increase of torque causes a reduction of angular velocity and vice versa. Large gear wheels (as a result of large demanded gear ratios) are avoided by introducing a countershaft and realizing the gear as a series of two gear wheel pairs (pre-transmission and post-transmission) with smaller gear wheel radii each. Two or three countershafts instead of one can be used to reduce the axial expansion of the gearbox.

Many modern gearboxes are so-called dual-clutch transmission systems. That is, there are two input shafts which can separately be rotated and clutched to the motor. Even numbered gears are assigned to one input shaft and odd numbered gears are assigned to the other input shaft. Thereby, changing gears becomes possible without interruption of traction. To save space, the two input shafts are realized as a long full shaft fitted inside a shorter hollow shaft.

With respect to gear wear and noise impact, it is advantageous to let the gear wheels always engage with each other. In that case, a design modification is required in order to prevent blockage of the transmission: The gear wheels are pivot-mounted on the shafts, i.e., they can rotate independently from each other. A power flow is established by coupling some gear wheels to their respective shafts using synchronization systems. Due to their high complexity, using as few of these systems as possible has a high priority. For instance, a sliding sleeve can be placed between two gear wheels and synchronize either one of them, but not simultaneously. Alternatively, selected gear wheels can be mounted fix onto a shaft to omit a synchronizer.

To simplify the placement and interaction of components, we can assume that gear wheels and sliding sleeves need approximately the same axial space: In this way, we can infer a discrete number of alignment planes in a gearbox of given expansion. A shaft and an plane together determine a position where a component can be placed. Gears that are placed on the same plane can potentially interact with each other.

The manual design procedure

The conventional design procedure of a gearbox is a multi-phase process chain. The planning department is confronted with the task to design a new gearbox for a vehicle. There may be early assessments of the available space and of the forces the gears will have to bear, but these plans may change in later development stages. In an abstract view, the planner has to pass through the following three planning stages:

Topology Some initial design choices like the number of shafts, the number of gears and the desired gear transmission ratios can be based on experience, empirical values or quick calculations. These decisions become more complex, if the same topology must be shared by multiple vehicle models: While being economically favorable, this approach may force the planner to make compromises on the performance of the gearbox in later development stages. For instance, fixed shaft distances are a restriction on the gear wheel radii and therefore on the possible transmission ratios.

Transmission In this stage, the requested transmission ratios have to be realized by placing gear wheels onto shafts. The planner has to choose gear wheel radii that not only lead to the right transmission ratio for each gear, but that also fit well (i.e. in a space-saving way) into the gearbox topology. An additional degree of freedom is the balance between pre-transmission and post-transmission. The height of the gearbox is a consequence of the gearbox topology as a whole. As a simplification for this stage, the gear wheels are considered as toothless, i.e., the planner expects that the wheels can be geared in a subsequent stage without too many issues.

Gearing, Efficiency and Misalignment After the topology has been fixed, a range of extended feasibility criteria has to be checked. Firstly, the gear wheels have to be endowed with teeth. The tooth number along the diameter of each wheel is an integer, so not all gear wheel combinations engage equally well. Secondly, the gear realizations and the gearing of the wheels influence the transmission efficiency of the gearbox. Thirdly, shafts bend as a side effect of the power flow. If they bend to much, the teeth become misaligned and cause increased wear and acustic noise.

In each stage, two things can happen: (a) the planner realizes that his decisions in earlier stages are not compatible with the requirements of the next stage, or (b) the outer requirements for the design are updated. In both cases, the planner has to go back to an earlier stage and incorporate the new information into the current design proposal. This process is iterated until the planner and the company can settle on a final design.

An automated design procedure

To improve the manual design process, we have to view the system as a whole. However, the extended feasibility criteria in the third stage of the design procedure are highly complex and cannot be modeled within the scope of this work. Therefore, we focus on the first two stages of the design process: the topology and the transmission. Our aim is to find optimal gearbox proposals in a short amount of time. On the one hand, this enables the planner to consider a large number of possible requirement changes in advance. On the other hand, the planner can focus on technical aspects in stage three instead of repeating the work in earlier stages.

3 A GEARBOX DESIGN MODEL

The aim of the following model is to find an optimal gearbox design within a given axial space and for demanded total transmission ratios. In the following, capital letters denote sets or parameters (i.e. the problem input), small letters denote indices or continuous decision variables, and greek letters denote binary variables. An overview of all decision variables is given in Table 1. The full model is shown on the next page. The constraints will be explained in order of appearance. Parameters are explained together with the constraints in which they occur.

Three sets characterize the topology of the gearbox. Let $G = \{-1, 1, 2, \dots\}$ denote the set of gears (including one reverse gear -1), let $P = \{1, 2, \dots\}$ denote the set of planes that fit into the axial space, and let $D = \{1, 2\}$ denote the set of exactly two drive shafts. A plane and a shaft together determine a position where a component can be placed.

The objective function combines three objective criteria: (i) minimize the height of the gearbox, (ii) minimize the number of sliding sleeves, and (iii) minimize the number of gear wheels on the input shaft, cf. Eq. (1).

Table 1: Decision variables of the mixed-integer nonlinear program.

Var.	Description	Domain
i_g	pre-transmission of gear g	$[I^{\min}, I^{\max}]$
j_d	post-transmission from drive shaft d to the output shaft	$[J^{\min}, J^{\max}]$
r_g	gear wheel radius on the input shaft belonging to gear g	$[R^{\min}, R^{\max}]$
s_g	gear wheel radius on the drive shaft belonging to gear g	$[S^{\min}, S^{\max}]$
t_p	gear wheel radius on the input shaft in plane p	$[R^{\min}, R^{\max}]$
$u_{p,d}$	gear wheel radius on drive shaft d in plane p	$[S^{\min}, S^{\max}]$
v_d	maximum gear wheel radius on drive shaft d	$[S^{\min}, S^{\max}]$
y_d	gear wheel radius on drive shaft d of the final drive	$[Y^{\min}, Y^{\max}]$
z	gear wheel radius on the output shaft of the final drive	$[Z^{\min}, Z^{\max}]$
$\xi_{g,p,d}$	indicator whether gear g is realized on drive shaft d in plane p	$\{0, 1\}$
$\gamma_{p,d}$	indicator whether any gear is realized on drive shaft d in plane p	$\{0, 1\}$
$\delta_{g,p}$	indicator whether gear g is realized on any drive shaft in plane p	$\{0, 1\}$
ζ_p	indicator whether any gear is realized on any drive shaft in plane p	$\{0, 1\}$
$\sigma_{p,d}$	indicator whether a sleeve is located on drive shaft d in plane p	$\{0, 1\}$
ϕ_p	indicator whether the input shaft is a full shaft in plane p	$\{0, 1\}$
a_d	distance between input shaft and drive shaft d	$[A^{\min}, A^{\max}]$
b_d	distance between drive shaft d and output shaft	$[B^{\min}, B^{\max}]$
c	distance between the drive shafts	$[C^{\min}, C^{\max}]$
h	pseudo-height of the gearbox	$[H^{\min}, H^{\max}]$

The exact height along the vertical vehicle axis is quite difficult to express, because the two drive shafts need not be horizontally aligned. Therefore, we use the pseudo-height h defined by Eq. (24) as approximation of the real height. The weighing parameters W_1 and W_2 control how the objective criteria are balanced. This decision is at the discretion of the planner. For our benchmarks, we used $W_1 \gg 1$ and $W_2 \ll 1$, i.e., we tolerate a larger gearbox to save a sliding sleeve, but not to save a gear wheel on the input shaft.

Eq. (2) determines the overall transmission ratio for each gear. Since these overall ratios are empirical values, we allow for small deviations of $5\,\%$ from the reference values in hope of achieving a more compact design. The interval $[K_g^{\min}, K_g^{\max}]$ gives the range of acceptable overall ratios of gear g. The sum expression identifies the matching post-transmission j_d for the pre-transmission i_g of gear g. Eqs. (3) and (4) relate the gear wheel radii with the pre-transmission and respectively post-transmission ratios. The sign function is needed to allow for the negative pre-transmission ratio i_{-1} of the reverse gear. Note that we do not need to include the radius s_{int} of the intermediate wheel that engages with the input shaft gear wheel and the reverse gear wheel, since $i_{-1} = (s_{-1}/s_{\text{int}}) \cdot (s_{\text{int}}/r_{-1})$.

Eqs. (5) to (8) specify relations between various gear assignment indicators. The most general indicator $\xi_{g,p,d}$ becomes active (equal to one) if and only if the gear g is established along the position given by plane p and drive shaft d. This position specifies the gear wheel that needs to be synchronized to establish the power flow. The auxiliary variables $\gamma_{p,d}$, $\delta_{g,p}$ and ζ_p each indicate if at least one $\xi_{g,p,d}$ over some set is active. Additionally, Eq. (6) ensures that each gear g is realized exactly once.

$$\text{minimize} \qquad h + W_1 \cdot \sum_{p\in P}\sum_{d\in D} \sigma_{p,d} + W_2 \cdot \sum_{p\in P} \zeta_p \tag{1}$$

subject to

$$K_g^{\min} \le i_g \cdot \sum_{d\in D} j_d \cdot \sum_{p\in P} \xi_{g,p,d} \le K_g^{\max} \qquad \forall g \in G \tag{2}$$

$$r_g \cdot i_g = s_g \cdot \operatorname{sign}(g) \qquad \forall g \in G \tag{3}$$

$$y_d \cdot j_d = z \qquad \forall d \in D \tag{4}$$

$$\delta_{g,p} = \sum_{d\in D} \xi_{g,p,d} \qquad \forall g \in G,\ p \in P \tag{5}$$

$$1 = \sum_{p\in P} \delta_{g,p} \qquad \forall g \in G \tag{6}$$

$$\gamma_{p,d} = \sum_{g\in G} \xi_{g,p,d}\,, \quad \zeta_p \ge \gamma_{p,d} \qquad \forall p \in P,\ d \in D \tag{7}$$

$$\zeta_p \le \sum_{d\in D} \gamma_{p,d} \qquad \forall p \in P \tag{8}$$

$$\gamma_{p,d} + \sigma_{p,d} \le 1\,, \quad \gamma_{p,d} \le \sum_{\substack{p'\in P\\ |p'-p|=1}} \sigma_{p',d}\,, \quad \xi_{-1,p,d} \le \gamma_{p,3-d} \qquad \forall p \in P,\ d \in D \tag{9}$$

$$\phi_p \ge \phi_{p+1} \qquad \forall p \in P,\ p < |P| \tag{10}$$

$$\delta_{g,p} \le \begin{cases} \phi_g & \text{if } g \text{ is odd} \\ 1-\phi_g & \text{if } g \text{ is even} \end{cases} \qquad \forall p \in P,\ g \in G, g > 0 \tag{11}$$

$$\delta_{-1,p} \le 1 - \phi_p \qquad \forall p \in P \tag{12}$$

$$r_g = \sum_{p\in P} t_p \cdot \delta_{g,p}\,, \quad s_g = \sum_{p\in P}\sum_{d\in D} u_{p,d} \cdot \xi_{g,p,d} \qquad \forall g \in G \tag{13}$$

$$t_p \ge R^{\text{full}} \cdot \phi_p + R^{\text{hollow}} \cdot (1-\phi_p) \qquad \forall p \in P \tag{14}$$

$$t_p \le R^{\text{full}} \cdot \phi_p + R^{\text{hollow}} \cdot (1-\phi_p) + R^{\max} \cdot \zeta_p \qquad \forall p \in P \tag{15}$$

$$u_{p,d} \ge S^{\min} \cdot (1 - \sigma_{p,d} - \gamma_{p,d}) + S^{\text{sync}} \cdot \sigma_{p,d} + S^{\min} \cdot \gamma_{p,d} \qquad \forall p \in P,\ d \in D \tag{16}$$

$$u_{p,d} \le S^{\min} \cdot (1 - \sigma_{p,d} - \gamma_{p,d}) + S^{\text{sync}} \cdot \sigma_{p,d} + S^{\max} \cdot \gamma_{p,d} \qquad \forall p \in P,\ d \in D \tag{17}$$

$$v_d \ge u_{p,d} \qquad \forall p \in P, d \in D \tag{18}$$

$$a_d \ge t_p + u_{p,d} + Q \cdot (1-\gamma_{p,d}) + 2 \cdot Q \cdot \xi_{-1,p,d} \qquad \forall p \in P,\ d \in D \tag{19}$$

$$a_d \le t_p + u_{p,d} + A^{\max} \cdot (1-\gamma_{p,d}) \qquad \forall p \in P,\ d \in D \tag{20}$$

$$a_d \ge R^{\text{hollow}} + y_d + Q\,, \qquad b_d = y_d + z \qquad \forall d \in D \tag{21}$$

$$c \ge \sum_{d\in D} u_{p,d} + Q \cdot \big(\sum_{d\in D} \gamma_{p,d} - \delta_{-1,p}\big) \qquad \forall p \in P \tag{22}$$

$$c \le \sum_{d\in D} u_{p,d} + C^{\max} \cdot (1 - \delta_{-1,p}) \qquad \forall p \in P \tag{23}$$

$$h \ge c + \sum_{d\in D} v_d\,, \quad h \ge 2 \cdot z \tag{24}$$

Eq. (9) (left) demands that at most one component may be placed on a position and Eq. (9) (middle) ensures that each gear can be synchronized by a neighbouring sleeve. If the reverse gear wheel is placed on a position, another gear wheel must be placed on the other drive shaft in the same plane, cf. Eq. (9) (right).

The input shaft is divided into a full shaft and a hollow shaft, signalled by the fullness indicator ϕ_p for each plane p. There must only be one transition from full shaft to hollow shaft, so these indicators need to have a monotoneous behaviour, cf. Eq. (10). Odd numbered gears are established on full shaft planes and even numbered gears are established on hollow shaft planes, cf. Eq. (11). The reverse gear is also placed on the hollow shaft, which allows for quick shunting movements without interruption of traction, cf. Eq. (12).

The variables r_g and s_g represent the gear wheel radii from the point of view of the gears. The variables t_p and $u_{p,d}$ represent the radii from the point of view of the positions inside the gearbox. Eq. (13) identifies variables with the same meaning. If there is no gear wheel on plane p, Eqs. (14) and (15) set the radius at this position to the input shaft radius (concretely, they select the radius R^{full} of the full shaft or the radius R^{hollow} of the hollow shaft).

Eqs. (16) and (17) determine the drive shaft radii $u_{p,d}$: If the synchronizer indicator $\sigma_{p,d}$ is active, the radius must equal the synchronizer radius S^{sync} (which we assume as fixed), and if no indicator is active, the radius at this position must equal the radius S^{min} of the drive shaft. The maximum radius v_d of gear wheels on the drive shaft d in Eq. (18) is needed later for computing the pseudo-height h.

Eqs. (19) to (23) determine the pairwise distances between shafts. The distance a_d between the input shaft and drive shaft d equates the sum of engaging gear wheel radii. When multiple gears share the same drive shaft, all engaging gear wheel pairs have to agree on the same shaft distance. If only one component is a gear wheel, there must be a gap of the tooth height Q between the gear and the sleeve or shaft. Between the input shaft gear wheel and a not engaging reverse gear, the gap must be at least two tooth heights. Finally, the distance a_d must be large enough to separate the final drive wheel from the hollow shaft, cf. Eq. (21) (left).

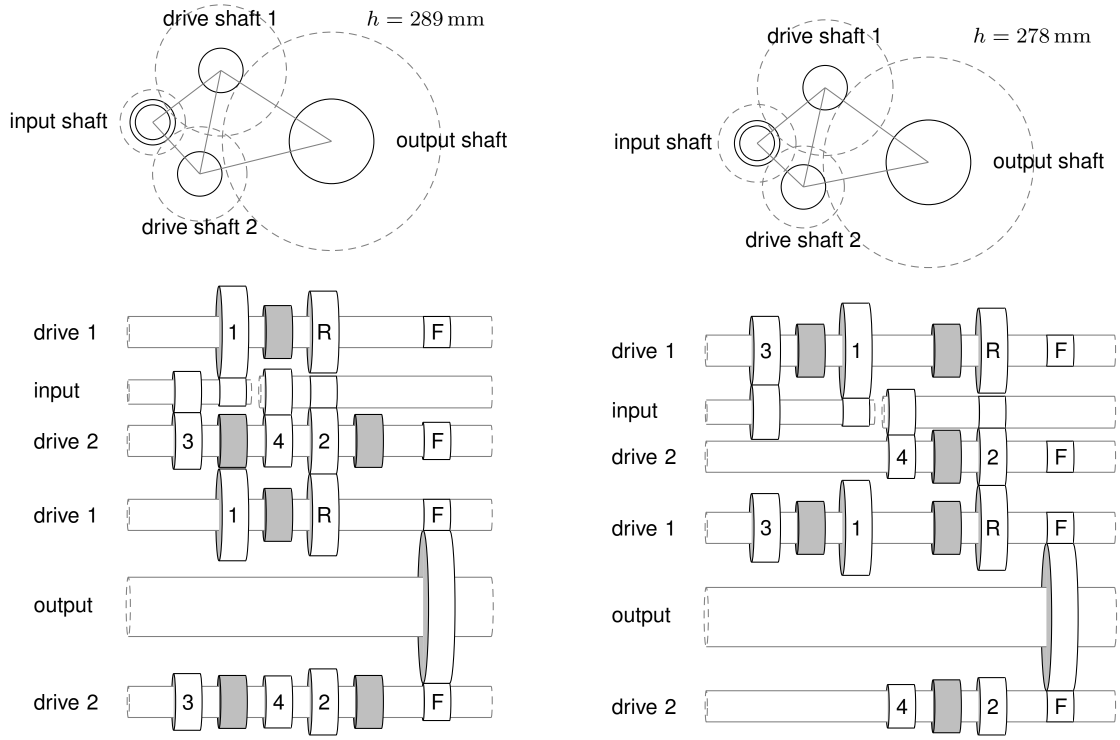
The distance b_d between the drive shaft d and output shaft depends on the final drive radii y_d and z. Between the two drive shafts, no gears wheels are allowed to engage except for the reverse gear wheel.

The pseudo-height h that enters the objective is the maximum of two values: Firstly, the drive shaft distance c plus the respective maximum radii on both shafts. Secondly, the diameter of the final drive gear wheel on the output shaft. The pseudo-height is a good approximation of the gearbox height if the two drive shafts are horizontally not too far apart.

4 BENCHMARKS AND RESULTS

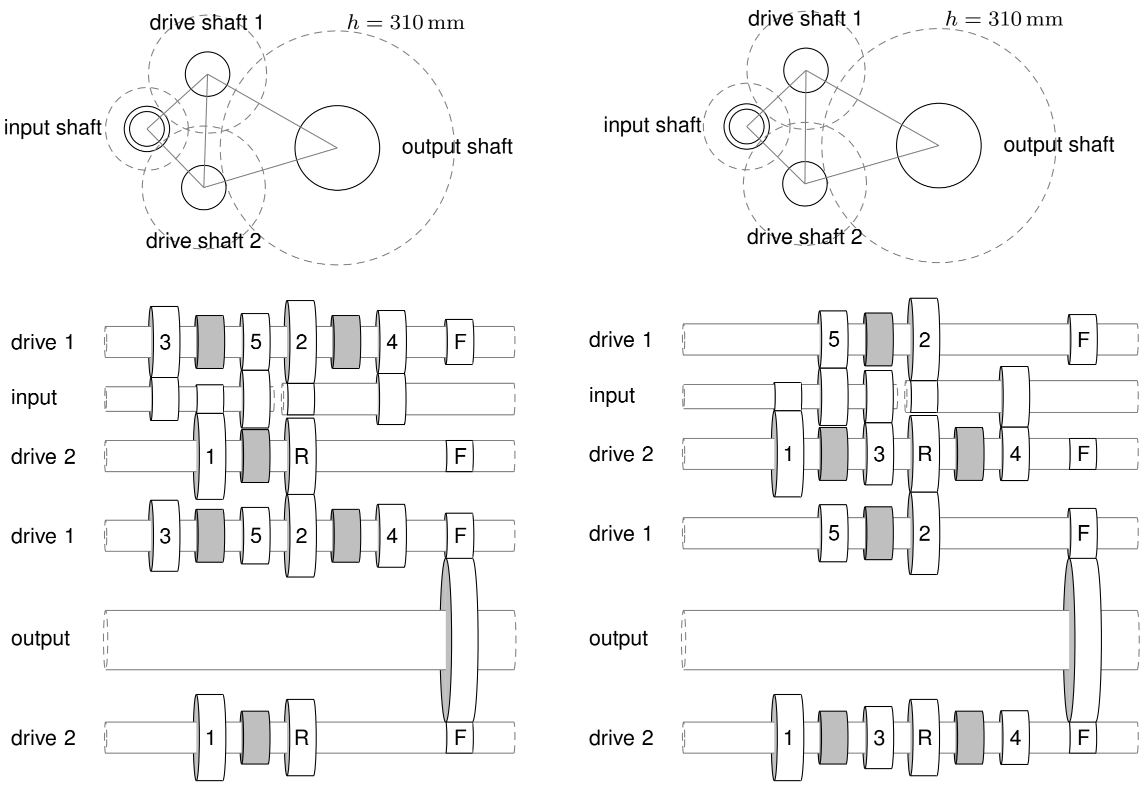
The mixed-integer nonlinear program (MINLP) has been implemented in the mathematical modeling framework *JuMP [3] (version 0.12.0)* in the programming language *Julia (version 0.4.3)*. We have chosen *Couenne [4] (version 0.5.6)*, a free global solver for nonconvex MINLP based on branch-and-bound and convex envelopes, and *SCIP [5] (version 3.2.1)*, a commercial open-source framework for Constraint Integer Programming, to solve the following instances.

As first example, we design a gearbox with four gears and one reverse gear. If we make only four planes available, the solvers return with the status *infeasible*, i.e, there is no imaginable gearbox design in four planes under the stated rule set. It turns out that there are possible designs for five and six available planes. Couenne needed $T^{\text{couenne}}_{4+1,5} = 13\,\text{s}$ and $T^{\text{couenne}}_{4+1,6} = 21\,\text{s}$ to prove optimality. SCIP was done in $T^{\text{scip}}_{4+1,5} = 5\,\text{s}$ and $T^{\text{scip}}_{4+1,6} = 9\,\text{s}$.



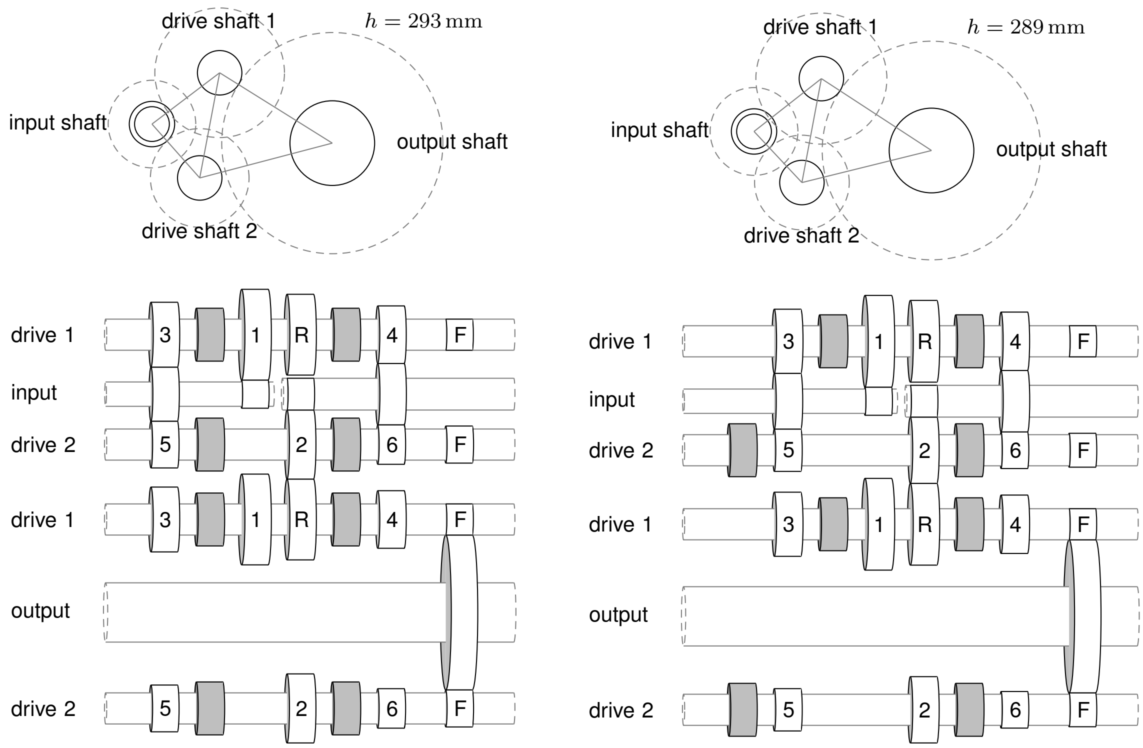
(a) Minimum axial expansion: five planes. (b) Additional axial space: six planes.

Figure 2: Optimal gearboxes with four plus one gears.



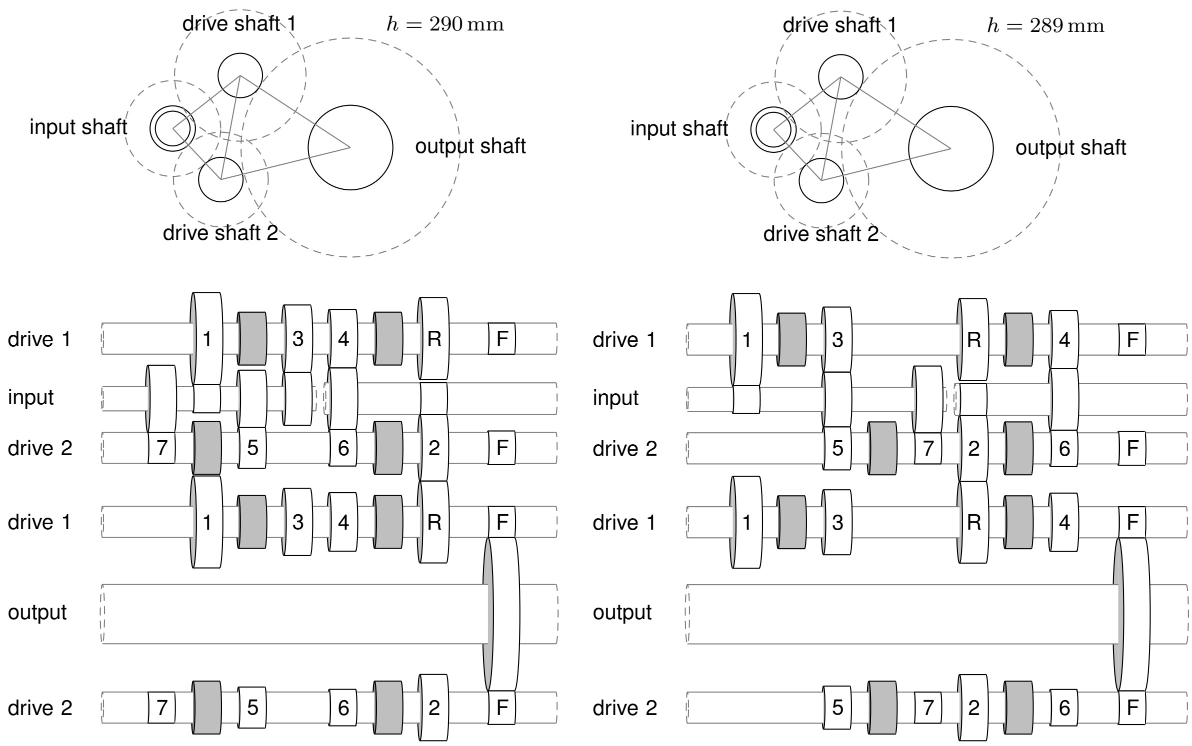
(a) Minimum axial expansion: six planes. (b) Additional axial space: seven planes.

Figure 3: Optimal gearboxes with five plus one gears.



(a) Minimum axial expansion: six planes. (b) Additional axial space: seven planes.

Figure 4: Optimal gearboxes with six plus one gears.



(a) Minimum axial expansion: seven planes. (b) Additional axial space: eight planes.

Figure 5: Optimal gearboxes with seven plus one gears.

Figure 2 shows a representation of the results. On the top, the gearbox is viewed in axial direction – relating to the gearbox in Figure 1, the viewpoint is on the right. The black circles depict the cross sections of the shafts and the gray lines represent distances that are given by decision variables (a, b and c). The dashed circles indicate the maximum gear wheel radius on each shaft. (They are allowed to overlap as long as these gear wheels do not share a plane.) On the top right, the optimal pseudo-height h is given.

On the bottom of each figure, we see a folding pattern of the gearbox topology. It shows the size and position of the gear wheels and sliding sleeves, the pairwise distances between the shafts, and the gear wheel engagements. The final drive gear wheels coupling drive shafts and output shaft are positioned on the far right and labeled with the letter F. The input shaft is separated into the full shaft on the left and the hollow shaft on the right.

In the presented model, we have not considered power flows along arbitrary many gear wheel pairs: Normal gears use two gear wheel pairs (pre-transmission and post-transmission) and the reverse gear uses three gear wheel pairs (the pre-transmission of a normal gear, the transmission to the reverse gear wheel and the post-transmission). Therefore, in these solutions the power flow of each gear takes the shortest route from the input shaft over the accordingly labeled gear wheel to the output shaft. In the folding pattern, the power flow *appears to jumps* from a drive shaft engaging with the input shaft to its identical copy engaging with the output shaft.

Next, we design gearboxes with five gears and one reverse gear. There is no design with only five planes available. The solutions for six and seven planes are depicted in Figure 3. The runtimes were $T^{\text{couenne}}_{5+1,6} = 28\,\text{s}$ and $T^{\text{coenne}}_{5+1,7} = 73\,\text{s}$ for Coeunne, and $T^{\text{scip}}_{5+1,6} = 5\,\text{s}$ and $T^{\text{scip}}_{5+1,7} = 11\,\text{s}$ for SCIP.

The effect of additional axial space is underwhelming: Both gearboxes have exactly the same pseudo-height and the additional plane is not even used. The gear wheels of both solutions are slightly permutated. This is a random effect from the solution procedure, because neither topology is *better* regarding our objective function.

Optimal gearboxes with six plus one and seven plus one gears are shown in Figure 4 and Figure 5. There is no design for a six plus one gearbox in five planes and neither a design for a seven plus one gearbox in six planes. The computations with Couenne required $T^{\text{couenne}}_{6+1,6} = 35\,\text{s}$ and $T^{\text{couenne}}_{6+1,7} = 168\,\text{s}$ for the six plus one gearboxes and $T^{\text{couenne}}_{7+1,7} = 436\,\text{s}$ and $T^{\text{couenne}}_{7+1,8} = 803\,\text{s}$ for the seven plus one gearboxes. The computations with SCIP required $T^{\text{scip}}_{6+1,6} = 21\,\text{s}$ and $T^{\text{scip}}_{6+1,7} = 37\,\text{s}$ for the six plus one gearboxes and $T^{\text{scip}}_{7+1,7} = 117\,\text{s}$ and $T^{\text{scip}}_{7+1,8} = 116\,\text{s}$ for the seven plus one gearboxes.

Again, the additional plane does not help in achieving a more space-efficient design. The difference of the millimeter values between the left and the right can be explained as numerical artifact. In the case of Couenne, we needed to increase the allowable fractional gap and the feasibility tolerance to compute the presented results. SCIP has stricter default tolerances than Couenne but did warn of a constraint violation in the order of 10^{-6}. These large instances seem to be numerically challenging. Instead of increasing solver tolerances even further, we think it would be more useful to search for improved model formulations.

Note that the gearboxes with five plus one gears (cf. Fig. 3) need *more* space than the gearboxes with one additional gear (cf. Fig. 4). This is an intended effect of the objective criteria weighing: Five plus one gears can be realized with only three sliding sleeves, but a fourth sleeve would have allowed for a more efficient packing.

5 CONCLUSIONS

- We presented a mixed-integer nonlinear program for the optimal design of gearboxes on the example of a dual-clutch transmissions.
- Optimal designs for four plus one gearboxes and up to seven plus one gearboxes have been presented. The computation times of both solvers in the range of seconds to minutes are quite reasonable.
- This paper only shows a first step to a practically useful gearbox design framework. For real-world use, we need to take further details, e.g. the gear teeth design and shaft bending, into account. We plan to integrate these factors into our model in further work.

REFERENCES

[1] R. Fischer, G. Jürgens, F. Küçükay, R. Najork, B. Pollak, Das Getriebebuch. Springer-Verlag, 2012.

[2] H. Naunheimer, B. Bertsche, G. Lechner, Fahrzeuggetriebe: Grundlagen, Auswahl, Auslegung und Konstruktion, 2007.

[3] M. Lubin, I. Dunning, Computing in Operations Research Using Julia, *INFORMS Journal on Computing*, **27**, No. 2, pages 238–248, 2015.

[4] P. Belotti, J. Lee, L. Liberti, F. Margot, A. Wächter, Branching and bounds tightening techniques for non-convex MINLP, *Optimization Methods and Software*, **24**, No. 4–5, pages 597–634, 2009.

[5] T. Achterberg, SCIP: Solving constraint integer programs, *Mathematical Programming Computation*, **1**, No. 1, pages 1–41, 2009.