Some Investigations of a Generalized Particle Method for Convection-Diffusion Equations

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ABSTRACT

A generalized particle method is considered for convection-diffusion equations. In Imoto–Tagami [1], the generalized particle method has been introduced as a class of particle methods, which can describes Smoothed Particle Hydrodynamics (SPH), Moving Particle Semi-implicit (MPS), and others, and the truncation error estimate has been established. Moreover, in Imoto–Tagami [2], error estimates of the generalized particle method for the Poisson equation and the heat equation have been introduced. Our goal is to construct of mathematical frameworks of particle methods, and this paper is regarded as the next step toward our goal.

At this step, the particle motions is considered, which play a key role in practical computational fluid dynamics with particle methods. In general, the particle motions cause particle distributions unevenness and numerical schemes instability. To overcome this difficulties, the Lagrange–Galerkin characteristic starategy (see, for example, Pironneau [4] and Notsu–Tabata [3]), is introduced into numerical schemes. The Lagrange–Galerkin characteristic starategy does not require particle redistributions in our numerical scheme and solves numerical instabilities of the scheme.

Some mathematical and numerical investigations are shown to confirm the effectiveness of our strategy.

References

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